Exercise 2

Economic growth: Theory and Empirical Methods, UC3M

Question 1: The Solow model, different from the Malthus model, suggests that changing policies, such as changing the savings rate, can lead to permanent differences in income per person, i.e., the model does not feature poverty traps.

1. Discuss the two fundamental ways in which the Solow model changes the Malthus model.
2. Here, we will change one of those two, the exogenous population growth. To that end, consider the Solow model without technological growth. Output is given by: $Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}$. Capital is accumulated according to $\dot{K}(t) = sY(t) - \delta K(t)$. Finally, the population growth rate is endogenous and given by $n(t) = \beta \ln\left(\frac{Y(t)}{L(t)}\right)$ with $\beta > 0$. Show that the growth rate of the capital-to-output ratio is given by

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{\dot{K}(t)}{K(t)} - (1-\alpha)\beta \ln\left(\frac{Y(t)}{L(t)}\right). \tag{1}$$

3. Write output per worker as a function of the capital-to-output ratio and replace it in the above equation.
4. Use the capital accumulation equation to find an expression for the growth rate of capital $\frac{\dot{K}(t)}{K(t)}$.
5. Assume a steady state exists with a constant capital-to-output ratio, $\dot{z}(t) = 0$. Write down the equilibrium condition for z^*.
6. Argue graphically that there still exists a unique steady state level of the capital-to-output ratio.
7. Analyze what happens to z^* when the savings rate, s, increases.
8. Analyze what happens to output-per-worker when the savings rate, s, increases.
9. Determine the growth rate of capital in steady state.
10. Explain why, different from the Malthus model, increases in output per worker can be permanent.

Question 2: Consider the Solow model with $\alpha = 0.3$, $s = 0.2$, $\delta = 0.04$, $n = 0.02$,and $g = 0.02$. The file *Students2.R* initializes the problem.

1. Compute the initial steady state capital-to-output ratio, z^*
2. Suppose the economy increases its savings rate to $s = 0.4$. Simulate the economy for 100 periods. In particular

- Assume that in period 0, the economy is in its old steady state with $A(0) = 1$.
- Recompute the new steady state capital-to-output ratio.
- Compute for each period the level of productivity using $A(t) = A(0)\exp(gt)$.
- Compute for each period the capital-to-output ratio using $z(t) = z^* + (z(0) - z^*)\exp(-\beta t)$.
- Compute for each period output per worker as $y(t) = z(t)^{\alpha/(1-\alpha)}A(t)$.
- Compute for each period the marginal product of capital as $MPK(t) = \frac{\alpha}{z(t)}$.
- Compute for each period the growth rate of the capital-to-output ratio as $\frac{\dot{z}(t)}{z(t)} = z(t)/z(t-1) - 1$.
- Compute for each period the growth rate of output as $\frac{\dot{Y}(t)}{Y(t)} = g + n + \frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)}$.

3. Plot the growth rates of the capital-to-output ratio over time.

4. The growth rate of capital is given by the net return per unit of investment per capital: $\frac{\dot{K}(t)}{K(t)} = s\frac{Y(t)}{K(t)} - \delta = \frac{s}{\alpha}MPK(t) - \delta$. The capital-to-output ratio is growing while this is larger than the growth rate of output. Plot the two relationships in a single graph.

5. Plot the marginal product of capital over time. Explain the intuition for the convergence of the capital-to-output ratio.